Letters

Design of Optimal Disturbance Rejection PID Controllers Using Genetic Algorithms

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***Abstract*—This paper presents a method to design an optimal disturbance rejection PID controller. First, a condition for disturbance rejection of a control system—H_∞-norm—is described. Second, the design is formulated as a constrained optimization problem. It consists of minimizing a performance index, i.e., the integral of the time weighted squared error subject to the disturbance rejection constraint. A new method employing two genetic algorithms (GAs) is developed for solving the constraint optimization problem. The method is tested by a design example of a PID controller for a servo-motor system. Simulation results are presented to demonstrate the performance and validity of the method.**

***Index Terms*—Disturbance rejection constraint, genetic algorithms, integral of the time weighted squared error performance index, optimization, PID controller.**

I. Introduction

The design of optimal disturbance rejection controllers with fixed structure, known in the control literature as mixed H_2/H_∞ problem [1], consists of minimizing a performance index subject to the disturbance rejection constraint of the type H_∞-norm. In [2], the design problem was formulated as a minimization of the integral of squared error subject to a disturbance rejection constraint. A hybrid method was used consisting of a genetic algorithm with binary coding for minimization of the integral of the squared error (ISE) and a numerical algorithm for evaluating the disturbance rejection constraint.

The ISE is employed often in optimal control system design. A disadvantage of the ISE performance index is that its minimization may result in a response with relatively small overshoot, but a long settling time because the ISE performance index weights all errors equally independent of time. An improvement of the step response can be obtained by using the integral of time weighted squared error (ITSE) performance index [3].

This paper extends and generalizes the method presented initially in [4] to solve the constrained optimization problem. It is based on two real-coded genetic algorithms (GAs). One GA is used for minimizing the ITSE performance index; another for maximizing the disturbance rejection constraint. The contribution of the present paper is twofold: 1) the formulation of the design of optimal disturbance controllers with fixed structure by minimization of the ITSE performance index, subject to a disturbance rejection constraint of the type H_∞-norm and 2) the application of two GAs to solve the optimization problem.

The remainder of the paper is organized as follows: Section II describes the controller design problem; Section III presents a design method based on GAs; Section IV gives a detailed design example and simulation results; and Section V presents some conclusions.

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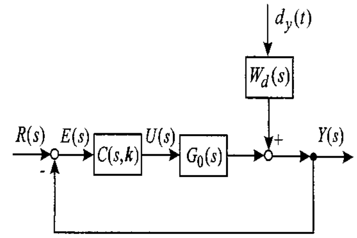

Fig. 1. Control system with disturbance.

II. Problem Description

In the case of conventional methods for controller design, it is assumed that the disturbance has a known form, i.e., a step or sinusoidal function. Using the H_∞-norm, the disturbance can be arbitrary, but the amplitude of the disturbance signal must be assumed limited. In the following, the condition for the disturbance rejection of a control system will be given. In the context of single-input–single-output linear time-invariant systems [1], consider the feedback control system shown in Fig. 1. The fixed-structure controller is described by a rational transfer function $C(s, \boldsymbol{k})$, where $\boldsymbol{k} = [k_1, k_2, \ldots, k_m]^{\mathrm{T}}$ designates the vector of the controller parameters. It is assumed that the plant to be controlled is described by the nominal transfer function $G_0(s)$ and is subject to an external disturbance $d_y(t)$ at its output.

A. Condition for Disturbance Rejection

Set $R(s) = 0$, then the disturbance rejection constraint can be described as

$$\max_{d_y(t)\in L_2} \frac{\|y\|_2}{\|d_y\|_2} = \left\| \frac{W_d(s)}{1 + C(s, \boldsymbol{k})G_0(s)} \right\|_\infty < \gamma$$

with $\gamma < 1$ as the desired rejection level.

$\|\cdot\|_\infty$ denotes the H_∞-norm, which is defined as

$$\|G(s)\|_\infty = \max_{w\in[0,\infty)} |G(jw)| \tag{1}$$

$W_d(s)$ is a weighting function consisting of a low-pass filter such as to reject the frequency response of the external disturbance $d_y(t)$.

By applying the H_∞-norm, the disturbance rejection constraint becomes

$$\begin{aligned}
&\left\| \frac{W_d(s)}{1 + C(s, \boldsymbol{k})G_0(s)} \right\|_\infty \\
&\quad = \max_{w\in[0,\infty)} \\
&\quad\cdot \left(\frac{W_d(jw)W_d(-jw)}{(1 + C(jw, \boldsymbol{k})G_0(jw))(1 + C(-jw, \boldsymbol{k})G_0(-jw))} \right)^{0.5} \\
&\quad = \max_{w\in[0,\infty)} (\alpha(w, \boldsymbol{k}))^{0.5}
\end{aligned}$$

where

$$\alpha(w, \boldsymbol{k}) = \frac{\alpha_z(w, \boldsymbol{k})}{\alpha_n(w, \boldsymbol{k})}$$

$$= \frac{W_d(jw)W_d(-jw)}{(1+C(jw,\boldsymbol{k})G_0(jw))(1+C(-jw,\boldsymbol{k})G_0(-jw))}.$$

Hence, the condition for disturbance rejection can be written in the frequency domain as

$$\max_{w\in[0,\infty)} (\alpha(w,\boldsymbol{k}))^{0.5} < \gamma.$$

The function $\alpha(w, \boldsymbol{k})$ can be expressed in the following form:

$$\alpha(w,\boldsymbol{k}) = \frac{\alpha_z(w,\boldsymbol{k})}{\alpha_n(w,\boldsymbol{k})} = \frac{\sum_{j=0}^{p}\alpha_{zj}(\boldsymbol{k})w^{2j}}{\sum_{i=0}^{q}\alpha_{ni}(\boldsymbol{k})w^{2i}}. \tag{2}$$

Both polynomials $\alpha_z(w, \boldsymbol{k})$ and $\alpha_n(w, \boldsymbol{k})$ have only even powers of w and the coefficients are functions of $\boldsymbol{k}$.

B. Design of Optimal PID Controller

The constraint for disturbance rejection is only a sufficient condition; therefore, it yields no information about the set point tracking of the control system. In many cases, a good closed-loop step response is desirable. In this paper, the controller design, i.e., the calculation of the vector $\boldsymbol{k}$, is obtained by minimizing a performance index. Let the nominal H_2 performance index be the ITSE given by

$$I = \int_0^\infty t\cdot(e(t))^2\,dt. \tag{3}$$

Using the Parseval theorem, the integral I can be defined in the frequency domain as

$$I = -\frac{1}{2\pi j}\int_{-j\infty}^{+j\infty}\frac{d}{ds}(E(s))\cdot E(-s)\,ds. \tag{4}$$

For a control system such as the one shown in Fig. 1 with $D_y(s) = 0$, the error signal $E(s)$ can be written as

$$E(s) = \frac{1}{1+C(s,\boldsymbol{k})G_0(s)}\cdot R(s) \tag{5}$$

where the set point $R(s)$ is a unit step signal.

The error signal $E(s)$ can be developed as a rational function

$$E(s) = \frac{D(s)}{A(s)} = \frac{\sum_{j=0}^{m} d_j s^{m-j}}{\sum_{i=0}^{n} a_i s^{n-i}}. \tag{6}$$

The condition that the integral in (4) will be finite can be satisfied if the degree m of the polynomial $D(s)$ is always smaller than the degree n of the polynomial $A(s)$. Furthermore, a_n can also be zero.

Inserting the error signal from (6) into (4) yields

$$I = -\frac{1}{2\pi j}\int_{-j\infty}^{+j\infty}\frac{d}{ds}\left\{\frac{\sum_{j=0}^{m} d_j s^{m-j}}{\sum_{i=0}^{n} a_i s^{n-i}}\right\} \cdot\frac{\left(\sum_{j=0}^{m} d_j(-s)^{m-j}\right)}{\left(\sum_{i=0}^{n} a_i(-s)^{n-i}\right)}\,ds. \tag{7}$$

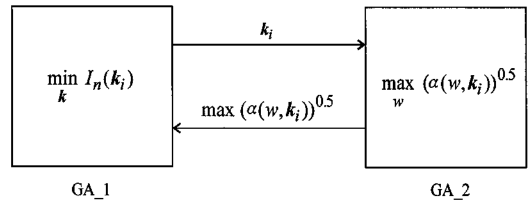

Fig. 2. Representation of the developed method using two GAs.

Equation (7) can be solved using the residue theorem. Closed solutions for I as functions of the coefficients a_i with $i = 0, \ldots, n$ and d_j with $j = 0, \ldots, m$ of the error signal $E(s)$ can be found in [3]. Since $E(s)$ contains the parameters of the controller (vector $\boldsymbol{k}$), the value of I for a system of nth order can be minimized by adjusting the vector $\boldsymbol{k}$ as follows:

$$\min_{\boldsymbol{k}} I_n(\boldsymbol{k}).$$

The evaluation of the performance index ITSE depends upon the stability of the closed-loop system. Consequently, the value of I_n is always positive, as it should be for stable systems. The stability and evaluation of I_n can be performed simultaneously. If the stability condition is not satisfied, then the computation of I_n can be terminated.

C. Design of Optimal Disturbance Rejection PID Controller

The design of the controller consists of the minimization of the ITSE performance index subject to the disturbance rejection constraint, as follows:

$$\min_{\boldsymbol{k}} I_n(\boldsymbol{k}) \qquad \text{subject to } \max_{w}(\alpha(w,\boldsymbol{k}))^{0.5} < \gamma.$$

The essence of the optimization lies in finding the vector $\boldsymbol{k}^*$, such that the ITSE performance index $I_n(\boldsymbol{k}^*)$ is a minimum and the constraint max $(\alpha(w, \boldsymbol{k}^*))^{0.5} < \gamma$ is satisfied. In the following, a solution is presented for this constrained optimization problem using real-coded GAs.

III. Genetic Algorithms and the Design Method

For the solution of the constrained optimization problem, two real-coded GAs are employed, i.e., GA_1 to minimize the performance index $I_n(\boldsymbol{k})$,and GA_2 to maximize the disturbance rejection constraint $\alpha(w, \boldsymbol{k})$, as depicted in Fig. 2. Initially, GA_1 is started with the controller parameters within the search domain as specified by the designer. These parameters are transferred then to GA_2, which is initialized with the variable frequency w.

GA_2 maximizes the disturbance rejection constraint during a fixed number of generations for each individual of GA_1. Next, if the maximum value of the disturbance rejection constraint is larger than γ, a penalizing value will be associated to the corresponding individual of GA_1. Individuals of GA_1 that satisfy the disturbance rejection constraint will not be penalized. In the evaluation of the fitness function of GA_1, individuals with higher fitness values are selected automatically and those penalized will not survive the evolutionary process.

For the implementation of the GAs, we used tournament selection, arithmetic crossover, and mutation [5].

A. Representation

In the real-coded representation, each individual is coded as a vector of floating-point numbers (a similar approach could be taken with evolution strategies or evolutionary programming [9]). For the design problem at hand, the parameters of the controller (vector $\boldsymbol{k}$) were coded in floating point and concatenated in an individual for

GA_1. For GA_2, an individual consists of only one gene (frequency w). The GAs were initialized randomly.

B. Fitness Function

An approach using penalty function [6] is employed to solve the constrained optimization problem.

Let the ITSE performance index be $I_n(\boldsymbol{k})$. Then the value of the fitness of each individual of GA_1 $\boldsymbol{k}_i (i = 1, \ldots, \mu_1)$ is determined by the evaluation function, denoted by $F_1(\boldsymbol{k}_i)$ as

$$F_1(\boldsymbol{k}_i) = -(I_n(\boldsymbol{k}_i) + P(\boldsymbol{k}_i)) \tag{8}$$

where μ_1 denotes the population size of GA_1. The penalty function $P(\boldsymbol{k}_i)$ is discussed in the following.

Let the disturbance rejection constraint be $\max(\alpha(w, \boldsymbol{k}_i))^{0.5}$. The value of the fitness of each individual of GA_2 $w_j (j = 1, \ldots, \mu_2)$ is determined by the evaluation function, denoted by $F_2(w_j)$ as

$$F_2(w_j) = \alpha(w, \boldsymbol{k}_i) \tag{9}$$

where μ_2 denotes the population size of GA_2.

The penalty for the individual $\boldsymbol{k}_i$ is calculated by means of the penalty function $P(\boldsymbol{k}_i)$ given by

$$P(\boldsymbol{k}_i) = \begin{cases} M_2, & \text{if } \boldsymbol{k}_i \text{ is unstable} \\ M_1 \cdot \max(a(w, \boldsymbol{k}_i)), & \text{if } \max(a(w, \boldsymbol{k}_i))^{0.5} > \gamma \; . \\ 0, & \text{if } \max(a(w, \boldsymbol{k}_i))^{0.5} < \gamma \end{cases} \tag{10}$$

If the individual $\boldsymbol{k}_i$ does not satisfy the stability test applied to the characteristic equation of the system, then $\boldsymbol{k}_i$ is an unstable individual and it is penalized with a very large positive constant M_2. Automatically, $\boldsymbol{k}_i$ does not survive the evolutionary process. If $\boldsymbol{k}_i$ satisfies the stability test, but not the disturbance rejection constraint, then it is an infeasible individual and is penalized with $M_1 \cdot \max a(w, \boldsymbol{k}_i)$, where M_1 is a positive constant to be adjusted. Otherwise, the individual $\boldsymbol{k}_i$ is feasible and is not penalized.

C. Method for Design of Optimal Disturbance Rejection PID Controller

The method can be summarized, as follows.

- Given the plant with transfer function $G_0(s)$, the controller with fixed structure and transfer function $C(s, \boldsymbol{k})$, and the weighting function $W_d(s)$, determine the error signal $E(s)$ and the disturbance rejection constraint $a(w, \boldsymbol{k})$.
- Specify the lower and upper bounds of the controller parameters.
- Set up GA_1 and GA_2 parameters: crossover probability, mutation probability, population size, and maximum number of generations.

It is more convenient to describe the method in the form of an algorithm.

Step 1: Initialize the populations of GA_1 $\boldsymbol{k}_i (i = 1, \ldots, \mu_1)$ and GA_2 $w_j (j = 1, \ldots, \mu_2)$, and set the generation number of GA_1 to $g_1 = 1$, where g_1 denotes the number of generations for GA_1.

Step 2: For each individual $\boldsymbol{k}_i$ of the GA_1 population, calculate the maximum value of $a(w, \boldsymbol{k}_i)$ using GA_2. If no individuals of the GA_1 satisfy the constraint $\max(a(w, \boldsymbol{k}_i))^{0.5} < \gamma$, then a feasible solution is assumed to be nonexistent and the algorithm stops. In this case, a new controller structure has to be assumed.

Step 3: Calculate the fitness value for each individual $\boldsymbol{k}_i$ of GA_1 by using (8) and (10).

Step 4: Select individuals using tournament selection and apply genetic operators (crossover and mutation) to the individuals of GA_1.

Step 5: For each individual $\boldsymbol{k}_i$ of the GA_1, calculate $\max(a(w, \boldsymbol{k}_i))^{0.5}$ using GA_2, as follows.

Substep a: Initialize the gene of each individual $w_j (j = 1, \ldots, \mu_2)$ in the population and set the generation number to $g_2 = 1$, where g_2 indicates the number of generations for GA_2.

Substep b: Evaluate the fitness of each individual by using (9).

Substep c: Select individuals using tournament selection and apply genetic operators (crossover and mutation).

Substep d: If the maximum number of generations of GA_2 is reached, stop and return the fitness of the best individual $\max(a(w, \boldsymbol{k}_i))$ to GA_1; otherwise, set $g_2 = g_2 + 1$ and go to substep b.

Step 6: If the maximum number of generations of GA_1 is reached, stop; otherwise, set $g_1 = g_1 + 1$ and go to step 3.

IV. Design Example

To illustrate the method, a detailed design example is presented in this section. Consider the control system shown in Fig. 1. The plant, a servomotor, is described by the following transfer function [2]

$$G_0(s) = \frac{0.8}{s(0.5s + 1)}. \tag{11}$$

The weighting function $W_d(s)$ [2] is chosen as

$$W_d(s) = \frac{1}{s + 1}. \tag{12}$$

The external disturbance is considered to be $d_y(t) = 0.1 \sin t$ and the disturbance attenuation level specified is $\gamma = 0.1$.

The controller $C(s, \boldsymbol{k})$ is described by the following transfer function:

$$C(s, \boldsymbol{k}) = k_1 + \frac{k_2}{s} + k_3 s. \tag{13}$$

The vector $\boldsymbol{k}$ of the controller parameter is given by

$$\boldsymbol{k} = [k_1, k_2, k_3]^{\mathrm{T}} = [k_p, k_i, k_d]^{\mathrm{T}}.$$

Because the plant, as described by (13), already contains an integral action, a controller with integral action (k_2) to control this plant is not necessary.

Assuming the input signal is a unit step, the error signal $E(s)$ is evaluated as follows:

$$E(s) = \frac{d_0 s^2 + d_1 s}{a_0 s^3 + a_1 s^2 + a_2 s + a_3} \tag{14}$$

with

$$\begin{aligned} d_0 &= 0.5,\ d_1 = 1,\ d_2 = 0,\ a_0 = 0.5, \\ a_1 &= 1 + 0.8k_3,\ a_2 = 0.8k_1,\ a_3 = 0.8k_2. \end{aligned}$$

The ITSE performance index $I_3(\boldsymbol{k})$ is shown in (15) at the bottom of the page [3]. The disturbance rejection constraint is calculated as

$$\alpha(w,\, \boldsymbol{k}) = \frac{\alpha_z(w,\, \boldsymbol{k})}{\alpha_n(w,\, \boldsymbol{k})} \tag{16}$$

with

$$\begin{aligned} \alpha_z(w,\, \boldsymbol{k}) &= w^4 + 0.25w^6 \\ \alpha_n(w,\, \boldsymbol{k}) &= 0.64k_2^2 + (-1.6k_2 - 1.28k_3k_2 \\ &\qquad + 0.64k_2^2 + 0.64k_1^2)w^2 \\ &\quad + (1 + 1.6k_3 + 0.64k_3 - 1.6k_i - 1.28k_3k_2 \\ &\qquad - 0.8k_1 + 0.64k_1^2)w^4 \\ &\quad + (1.25 + 1.6k_3 + 0.64k_3^2 - 0.8k_1)w^6 \\ &\quad + 0.25w^8. \end{aligned}$$

The controller parameter was searched in the following bounds [2]

$$k_1 = [0,\, 30]; \quad k_2 = [0,\, 30]; \quad k_3 = [0,\, 30].$$

The GA parameters were kept constant for all the simulations with crossover probability $p_{c1} = p_{c2} = 0.35$, mutation probability $p_{m1} = p_{m2} = 0.02$, population size for GA_1 $\mu_1 = 100$, population size for GA_2 $\mu_2 = 50$, penalty constant $M_t = 1\,000\,000$, penalty constant $M_s = 100$, maximum number of generations for GA_1 $g_{1\max} = 100$, and maximum number of generations for GA_2 $g_{2\max} = 50$. The values for crossover probability and mutation probability follow standard implementations in the literature [5].

The proposed method of using a real-coded GA, as described in the previous section, has been applied to the design of the PID controller. The convergence of the minimization of the ITSE performance index $I_3(\boldsymbol{k})$, subject to the disturbance rejection constraint $\max(a(w,\, \boldsymbol{k}_i))^{0.5}$ by using GA_1 for the best individual during the first 20 generations, is shown in Fig. 3. The minimum value $I_3(\boldsymbol{k}^*) = 0.000668$ is achieved in 11 generations and the corresponding best individual, i.e., the vector of controller parameters, yields $\boldsymbol{k}^* = [29.992,\, 0.00001,\, 28.3819]^{\mathrm{T}}$. The result reveals that optimization using the GA yields $k_2 = k_i = 0$. This confirms the suitability of GA as optimization method because there is no need for an integral action to control the plant as explained.

The calculation of the maximum value of the disturbance rejection constraint for the optimal vector of controller parameters $\boldsymbol{k}^*$ by using GA_2 is shown in Fig. 4. The maximum value is $(\alpha(w,\, \boldsymbol{k}^*))^{0.5} = 0.024\,60$. Because this value is smaller than γ, it means that $\boldsymbol{k}^*$ represents a feasible individual. Therefore, the condition for disturbance rejection is satisfied. The results obtained using the ITSE performance index show an improved behavior as compared to those employing the ISE performance index [2].

The performance of the control system in Fig. 1, utilizing a controller design based on the proposed method, is tested by closed-loop step response for two cases: 1) without disturbance and 2) with disturbance acting on the output of the plant. In Fig. 5, the step response of the control system with the controller parameters (vector $\boldsymbol{k}^*$) is shown

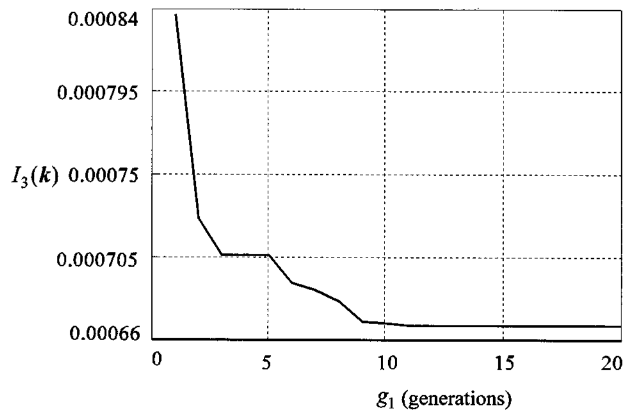

Fig. 3. Convergence of the minimization of the ITSE performance index $I_3(\boldsymbol{k})$ subject to the disturbance rejection constraint $\max(\alpha(w,\, \boldsymbol{k}))^{0.5} < \gamma$.

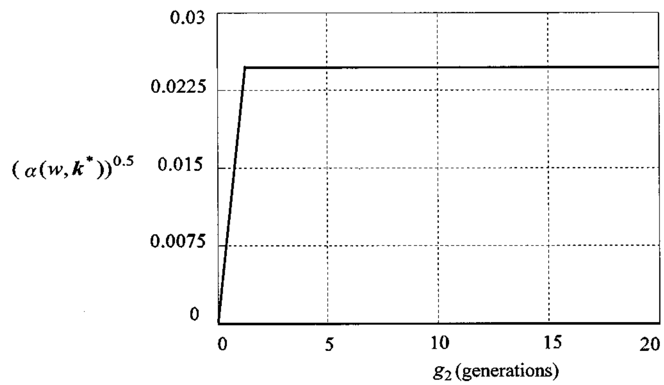

Fig. 4. Calculation of the maximum value of the disturbance rejection constraint $\max(\alpha(w,\, \boldsymbol{k}^*))^{0.5}$.

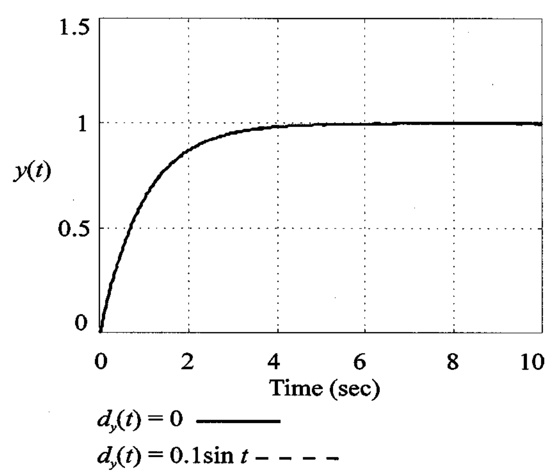

Fig. 5. Unit step response with a sinusoidal disturbance.

$$I_3(\boldsymbol{k}) = \frac{d_2^2}{4a_3^2} - \frac{d_0d_1 + d_1d_2\,\dfrac{a_1}{a_3}}{2(a_1a_2 - a_0a_3)} + \frac{d_0^2(a_2^2 + a_1a_3) + (d_1^2 - 2d_0d_2)(a_0a_2 + a_1^2) + \dfrac{d_2^2}{a_3}\,(a_0^2a_3 + a_1^2)}{2(a_1a_2 - a_0a_3)^2} \tag{15}$$

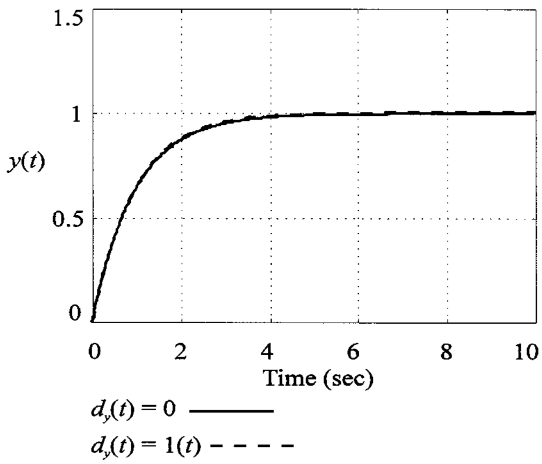

Fig. 6. Unit step response with an unit step disturbance.

for $d_y(t) = 0$ and sinusoidal disturbance $d_y(t) = 0.1 \sin t$. Fig. 6 shows the step response of the control system with the controller parameters (vector $\boldsymbol{k}^*$) for $d_y(t) = 0$ and unit step disturbance $d_y(t) = 1(t)$.

The closed-loop step responses present no overshoot and the settling time is about 4 s for a tolerance of $\pm 2\%$ of the set-point amplitude. It is observed from Figs. 5 and 6 that the closed-loop step response for the plant with either sinusoidal disturbance $d_y(t) = 0.1 \sin t$ or unit step disturbance $d_y(t) = 1(t)$ presents almost no difference compared to the nominal case, i.e., without disturbance. Therefore, the disturbance acting on the plant output has little influence on the step response.

The good step response of the control system with disturbance is due to the maximum value of the constraint for disturbance rejection $(\alpha(w, \boldsymbol{k}^*))^{0.5} = 0.024\,60$, which is small. In this way, the influence of the disturbance can be limited significantly. The results obtained clearly show the effectiveness of the proposed method in the design of an optimal disturbance rejection controller with fixed structure, for the case of a PID. The same technique also has been employed with very good results to design optimal robust controllers with fixed structure [7].

V. Conclusion

In this paper, a method is presented to design an optimal disturbance rejection controller with fixed structure, known in the literature as the mixed H_2/H_∞ problem. The design problem is formulated as an optimization problem with constraint of type H_∞-norm. A technique based on two real-coded GAs with appropriate operators and a penalty function is developed for the solution of the constrained optimization problem. One GA is used to minimize the ITSE performance index, and the other GA calculates the maximum value of the disturbance attenuation constraint. The method is illustrated by a design of a PID controller. The performance of the control system with a disturbance acting on the plant presents almost no difference, compared to the nominal case.

The advantages of the method can be summarized as follows: 1) there are no restrictions concerning the objective function or the constraint; 2) it was demonstrated by the application example that GAs are capable to find a very suitable solution in few generations; 3) the difficulties of implementation are very small, because GAs are very easy to implement; and 4) for each controller design problem, it is only necessary to define the fitness functions and the penalty function. Another objective of the paper is to show the control engineer how to utilize an intelligent method based on evolutionary computation for the design of PI/PID controllers widely accepted in automated environments [8].

Further developments of the new method are possible in many ways. For example, input disturbances can be taken in account. For this purpose, it is necessary to derive the condition for disturbance rejection. Alternatively, one may take other design goals in consideration as, for example, a constraint limiting the amplitude of the control signal. A variety of problems in different fields of research and application can be formulated similarly. Hence, it is reasonable to expect that the proposed method, with some necessary alterations, also could be applied to solve these problems.

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